Spin Diffusion Coefficient of A Phase of Liquid ^3He at Low Temperatures and Stability of Half Quantum Vortex

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We theoretically investigate the spin diffusion coefficient tensor in the A phase of liquid ^3He in term of quasiparticle life-time by using the Kubo formula approach at low temperatures. In general, the coefficient is a fourth rank tensor for the anisotropic states and can be defined as a function of both normal component of spin-current and magnetization. The quasiparticle life-time is obtained by using the Boltzmann equation. We find that components of the spin diffusion coefficient are proportional to T^{-2} at low temperatures. The normal components of spin current, hence, are strongly diffusive and one can ignore the contribution of these components to the stability of half quantum vortices $(HQVs)$ in the equal-spin-pairing of ^3He-A state. Hence to make a stable HQV, it is enough for one to consider weak interaction plus the effects of Landau Fermi liquid.

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1. INTRODUCTION

After the discovery of new phases of ^3He , a large amount of theoretical effort has been done on the spin collective excitations[1,2]. One view to the liquid phase of ^3He contains two normal and superfluid parts which at 3×10^{-3} Kelvin the superfluid part starts to be occurred mostly in triplet pairing[1,2]. The spin triplet pairing in the superconductor compound Sr_2RuO_4 bellow 1.5 Kelvin is observed experimentally[3] like the A phase of superfluid ^3He . The main similarity between ^3He-A and Sr_2RuO_4 phases is that only Cooper's pairs correspond to pair spin projections $S_z = 1$ and -1 or equal-spin-pairing (ESP) in these phases. The normal state properties of Sr_2RuO_4 are obtained in terms of a quasi two dimensional Fermi liquid quantitatively[3] whereas ^3He-A is a three dimensional Fermi liquid. This discrepanting manifest itself on the structure of the gap energies of them. The triplet pairing contains particles with the same spin directions that phenomenon leads to spin current. In two fluid model, this spin current includes normal and condensate (Cooper's pairs) components. The spin current of Cooper's pairs leads to interesting and important phenomena such as $HQVs$ in the ESP state. Unlike common vortices, the half-quantum vortices contain half-integer multiplications of the flux quantum $\Phi_0 = hc/2e$.

Vakaryuk and Leggett[4] represented a different analysis in the equal-spin-pairing superfluid state for a HQV. They used a BCS like wave function with a spin-dependent boost and predicted that in the stability of HQV an effective Zeeman field exists. In the thermodynamic stability state, the effective Zeeman field produces a non-zero spin polarization in addition to the polarization of external magnetic field. In their theory the effect of diffusive flow of the normal component did not take in to account. In this paper we show that only at very low temperature this is the case and in some limits of temperature the normal component might be not diffusive and take part in stability of HQV.

2. THEORETICAL APPROACH

Our formulation on spin diffusion coefficient is mainly concentrated on the Kubo formula approach[5]. For the A phase of ^3He superfluid it is briefly written in the following.In general, the spin diffusion D is a fourth rank tensor which may be explained by the relation:

$$\mathbf{J}^{(n)}_{i\alpha} = \sum_{\beta jl} D^{\alpha\beta}_{ij} (\frac{\partial}{\partial x_\beta})(\mathbf{S}_j - \chi_{jl}\mathbf{H}_l), \tag{1}$$

where $\mathbf{J}_n, \mathbf{S}, \mathbf{H}$ and χ are the spin current dyadic of the normal component, the magnetization, the static magnetic field, and the static magnetic susceptibility tensor, respectively. For the moment the dipole forces are ignored and according to the conservation law one can write:

$$(\frac{\partial \mathbf{S}_i}{\partial t}) + (\frac{\partial}{\partial x_\alpha})\mathbf{J}_{i\alpha} = 0, \tag{2}$$

in which the components of the total spin current $\mathbf{J}$ are

$$\mathbf{J}_{i\alpha} = \mathbf{J}^{(n)}_{i\alpha} + \mathbf{J}^{(s)}_{i\alpha} \quad \textbf{and} \quad \mathbf{J}^{(s)}_{i\alpha} = (\frac{1}{2m})\rho^{\alpha\beta}_{ij}\Omega_{j\beta}, \tag{3}$$

here $\Omega_{j\beta}$ is the component of the spin superfluid velocity dyadic[5]. The dynamic susceptibility tensor is defined by:

$$\mathbf{S}(\mathbf{k},\omega) = \sum_j \chi_{ij}(\mathbf{k},\omega)\mathbf{H}_j(\mathbf{k},\omega). \tag{4}$$

By substituting Eq. (3) and Eq. (1) in Eq. (2), and to compare with Eq. (4) is obtained:

$$\lim_{\omega\to 0}\lim_{k\to 0}\frac{\omega}{k^2}\chi''_{il}(\mathbf{k},\omega) = \sum_{\alpha\beta j} D^{\alpha\beta}_{ij}\widehat{k}_\alpha\widehat{k}_\beta\chi_{jl}, \tag{5}$$

here $\chi''_{il}(\mathbf{k},\omega)$ is the imaginary part of the spin dynamic susceptibility $\chi_{il}(\mathbf{k},\omega)$. Following Kadanoff and Martin's procedure on the imaginary part of the dynamic susceptibility with the anti-commutator of the magnetization in the normal phase one may generalize their result to the anisotropic superfluid states[5]. In the absent of the dipole interaction one may write:

$$\langle\{S_i(\mathbf{r},t),S_j(\mathbf{r}',t')\}\rangle = \int\frac{d\omega}{\pi}\int\frac{d\mathbf{k}}{(2\pi)^3}coth(\frac{\beta\omega}{2})\chi''_{ij}(\mathbf{k},\omega)\times\exp[i\mathbf{k}.(\mathbf{r}-\mathbf{r}')-i\omega(t-t')]. \tag{6}$$

With suppose that only the spin current associated with the normal component takes part in the diffusive flow, the effects of the superfluid component being negligible. By using Eq. (6) and Eq. (2) in Eq. (5), is obtained

$$D^{\alpha\beta}_{ij}\chi_{jl} = \frac{1}{4}\beta\lim_{\omega\to 0}\int_0^\infty dr\int dt\, e^{i\omega t}\langle\{\mathbf{J}^{(n)}_{i\alpha}(t),\mathbf{J}^{(n)}_{l\beta}(0)\}\rangle, \tag{7}$$

$\langle\rangle$ represents the expectation value in the equilibrium ensemble. By introducing the correlation function $K(\tau)$ as $K^{\alpha\beta}_{ij}(\tau) = \langle T_\tau\{\mathbf{J}^{(n)}_{i\alpha}(\tau),\mathbf{J}^{(n)}_{j\beta}(0)\}\rangle$ in which the time τ varies between $-\beta$ to $+\beta$. The spin diffusion coefficient D may be written in terms of the correlation function[5];

$$D^{\alpha\beta}_{ij}\chi_{jl} = \frac{1}{4}\lim_{\omega\to 0}\frac{ImK^{\alpha\beta}_{il}(\tau)(k=0,i\omega_l\to\omega+i\eta)}{\omega}, \tag{8}$$

where

$$\begin{aligned}K^{\alpha\beta}_{ij}(\omega_n,\mathbf{k}) = \frac{\mu^2}{8m^2}\int\frac{d^3\mathbf{p}}{(2\pi)^3}\frac{1}{\beta}\sum_m e^{\xi_m\eta}(k_\alpha+2P_\alpha)+(k_\beta+2P_\beta)\times\\ Tr[\alpha^i G(\mathbf{k}+\mathbf{P},\omega_n+\xi_m)\alpha^j G(\mathbf{P},\xi_m)].\end{aligned} \tag{9}$$

in which α^i are the Pauli matrices in the four-dimensional representation. After a bit of algebra Eq. (8) can be written as:

$$\begin{aligned}D^{\alpha\beta}_{ij}\chi_{jl} = \frac{\beta\chi_n V_F^2(1+\frac{1}{4}Z_0)}{32}\int d\xi\frac{d\Omega}{4\pi}\frac{d\omega}{2\pi}\widehat{p}_\alpha\widehat{p}_\beta\sec h^2(\frac{\beta\omega}{2})\times\\ Tr\{\alpha^i[G(\mathbf{P},\omega^+)-G(\mathbf{P},\omega^-)]\alpha^l[G(\mathbf{P},\omega^+)-G(\mathbf{P},\omega^-)]\}.\end{aligned} \tag{10}$$

For the ^3He-A phase, the Green's function may be written as

$$G(p,\omega) = \frac{\omega Z_p(\omega)+\xi\rho_3\times 1 - Z_p(\omega)\Delta(\alpha.\mathbf{d})\sigma_2\times\rho_2}{\omega^2 Z_p^2(\omega)-\xi_p^2-Z_p^2(\omega)\Delta^2(\Omega)}, \tag{11}$$

where $Z_p(\omega)$ is the re-normalization function. We define $\mathbf{d}\equiv -\frac{1}{2}i\sum_{\alpha,\beta}(\sigma_2\sigma)_{\alpha\beta}\Delta_{\alpha\beta}$ and use $G(p,\omega)$ for obtaining $D^{\alpha\beta}_{ij}\chi_{jl}$;

$$D^{\alpha\beta}_{ij}\chi_{jl} = \beta\chi_n V_F^2(1+\frac{Z_0}{4})\int\frac{d\Omega}{4\pi}\widehat{p}_\alpha\widehat{p}_\beta\int_\Delta^\infty d\omega\frac{\sec h^2(\frac{\beta\omega}{2})}{8Z_2(\omega^2-\Delta^2)^{\frac{3}{2}}}\times[(2\omega^2-\Delta^2)\delta_{il}+\Delta^2 d_i d_l], \tag{12}$$

here Z_2 is the imaginary part of $Z_p(\omega)$ and is simply handled by using the poles of the single-particle Green's function to define the quasiparticle energy and lifetime. Hence to lowest order in the imaginary parts we may write

$$Z_2 = \frac{EZ_1(E)}{2\tau(E,T)(E^2-\Delta^2)}. \tag{13}$$

Therefor, the spin diffusion coefficients for the ^3He-A phase are written as

$$D_{ij}^{\alpha\beta}\chi_{jl}=\chi_n V_F^2(1+\frac{Z_0}{4})\int\frac{d\Omega}{4\pi}\widehat{p}_\alpha\widehat{p}_\beta\int_0^\infty d\xi\sec h^2(\frac{\beta E}{2})\times\frac{\beta\tau(E,T)}{4Z_1(E)}[2\delta_{il}-\frac{\Delta^2}{E^2}(\delta_{il}-d_i d_l)]. \quad (14)$$

The above formula has been obtained with the condition $\omega_L\tau_D<1$ where ω_L is the Larmor frequency and τ_D is the spin diffusion lifetime. As to the experimental values of ω_L[6] this condition is fulfill even at very low temperatures. Hence, we may compute $\tau(E,T)$ at low temperatures. In this limit of temperature only the quasiparticles which are located at the nodes of the gap energy in the ^3He-A state take part to scattering processes and we may write[7]

$$\tau^{-1}(E,T)=(\frac{\pi\theta_m^2}{256\varepsilon_F})A_s^2\frac{[(\pi k_B T)^2+E^2]}{[1+\exp(-\frac{E}{k_B T})]} \quad (15)$$

where[8] $\theta_m\simeq\frac{\pi k_B T}{\Delta(0)}$,$A_s=\sum_l S_l$,$S_l=A_l^s-3A_l^a$ and $A_l^{s,a}=\frac{F_l^{s,a}}{[1+\frac{F_l^{s,a}}{(2l+1)}]}$, where $F_l^{s,a}$ are the Landua's parameters[7].

Finally, the spin diffusion coefficient in ^3He-A phase at low temperatures are $D_{zz}^{zz}=D_{xx}^{zz}\simeq 3D_{yy}^{zz}=C/T^2$ in which $C\simeq\frac{16V_F^2(1+F_0^a)\varepsilon_F\hbar}{A_s^2\pi^2k_B^2}$.

As it can be understood at low temperatures, the spin diffusion coefficients increase as T^{-2} and the normal components of spin current are strongly diffusive and one can ignore the contribution of these components to the stability of HQV.

3. CONCLUSIONS

In conclusion, we have investigated the temperature dependance of spin diffusion coefficient of superfluid ^3He-A at low temperatures. We have employed the Kubo approach and derived the spin-diffusive coefficient of A phase of the Liquid ^3He in term of quasiparticle life-time. The quasiparticle life-time is obtained by using the Boltzmann equation. We have found that the spin-diffusive coefficients at low temperatures regime are proportional to $1/T^2$. The finding suggests strongly diffusive normal components of spin-current and then weak contribution of them to the stability of the half-quantum vortex state. Our work of agreement with the assumption is in the recent work by Vakaryuk and Leggett[4]. The authors study the half-quantum vortex phenomenon in the absence of normal components of spin-current at $T=0$ and show the possibility of formation of the half-quantum vortices in the equal-spin paring of Landau Fermi liquid. The normal components of spin-current are important quantities which might generate important influences and our findings show good agreement with their assumption regarding the spin-current.

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